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FINITE ELEMENT ANALYSIS OF GFRP BOX CANTILEVER BEAMS

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ABSTRACT

A finite element model was developed to trace the Static response of advanced composite thin walled beams made of Glass Fiber Reinforced Plastics (GFRP). The model is a four node rectangular layered element with six degrees of freedom per node. The element stiffness matrix for the model includes the effect of coupling between membrane and bending actions. The developed model has the capability of taking an arbitrary number of layers of different material properties for the analysis of a given section. To demonstrate the accuracy and convergence of the program, the results were compared with those obtained by others in the literature. Stable convergence and good accuracy were obtained. The effects of material anisotropy, fibers orientation, fiber volume fraction, coupling between membrane and bending actions and layers transposition on the deflection of thin-walled GFRP box cantilever beams were evaluated.

INTRODUCTION

Advanced composites or specifically fiber reinforced plastics (FRP) laminates are being increasingly used in structural applications. FRP composites are mainly made of reinforcing materials such as fiber glass, carbon fibers and aramid fibers and matrix (resin) materials such as thermosetting or thermoplastic materials [1]. The mechanical properties of composites vary significantly with the type and orientation of the constituent resins and fibers [2]. Compared to standard construction materials, composite materials present many advantages, e.g., light weight, corrosion resistance and electromagnetic transparency. Most prominent is the

tailoring the material for each particular application [3]. The high initial cost of omposite materials is offset by their maintenance free performance. The low , self colored fiber reinforced polyester pultruded components offered many for the construction of light weight structures [4]. The design of structural manufactured from composite materials would initially involve the selection of the e materials, fiber orientation, volume fraction of fibers in resin and manufacture or the composite, each item will have a particular influence upon the mechanical f strength and stiffness of the final component [5]. To date, however, these ve been rarely used in civil engineering structures in Egypt. This is probably the complexities arising from the nature and structure of these materials which nalysis and design procedures [6].

of FRP sections is normally based on the layered approach, which deals with the inated composites where each lamina has a different orientation of fibers [7 and 8]. ethods for the analysis of thin plates and thin-walled beams made of laminated ections were developed earlier [9 and 10]. In this research, a rectangular element ed in order to study the theoretical behavior of advanced composite GFRP thin- s. A parametric study was carried out using the developed finite element model in stablish the influence of variation of moduli ratio (material anisotropy), fiber angle, fiber-volume fraction ratio, V_f, membrane-bending coupling and layers on the tip deflection of thin-walled GFRP box cantilever beams.

TION OF A RECTANGULAR ELEMENT MODEL

acement Relations for GFRP Composite Sections

ng to the classical lamination theory [11], the definite shape for composite material ated (multi-layered) section made up of several individual layers (laminae) in each he fibers are oriented in a matrix (see Figure 1). The strain-displacement relations at a distance z from the middle surface of the laminated composite section are

$$[\varepsilon] = [\varepsilon^\circ] - z[k] \qquad (1)$$

$[\varepsilon^\circ]$ contains the normal and shearing strains of the reference surface and,

$[k]$ contains the changes in curvature and angle of twist of the reference surface during deformation.

$$[\varepsilon^\circ] = \begin{bmatrix} \varepsilon^\circ_x \\ \varepsilon^\circ_y \\ \gamma^\circ_{xy} \end{bmatrix} = \begin{bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ \partial u / \partial y + \partial v / \partial x \end{bmatrix} \qquad (2\,a)$$

and

$$[k] = \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2(\partial^2 w / \partial x\, \partial y) \end{bmatrix} \qquad (2\,b)$$

In which ($\partial u/\partial x$, $\partial v/\partial y$,) are the first derivatives of the tangential mid plane displacements, u, v, w is the deflection of the cross section and ($\partial^2 w / \partial x^2$, $\partial^2 w / \partial y^2$, ..) are the second order derivatives of the deflection, w.

Force-Deformation Relationship

Timoshenko, and Woinowsky-Krieger [12] reported that the resultant in-plane forces and bending moments for isotropic plates may be obtained by integration across the thickness of the plate (See Figure 2). By applying Hooke's law for the stress-strain relationship in two dimensional layered system [3] and substituting into Equation (2), the final form of the force deformation relationship of a composite section is written as:

$$\begin{bmatrix} [N] \\ [M] \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix} [\varepsilon^\circ] \\ [k] \end{bmatrix} \qquad (3)$$

Where

$[N]$ = Membrane forces matrix.

$[M]$ = Bending stress resultants.

$[A]$, $[D]$ and $[B]$ are the effective membrane, bending and coupling stiffness matrices of a layered section, respectively.

erivation of the force-deformation relationship along with the matrices A, B, and D is sewhere [3]

ment Formulation

ular thin element of uniform thickness (t) with a local coordinates system o-x-y-z o-x and o-y are paralleled to the sides of the element and positive z-direction is ls, is considered in the analysis The origin is taken at the center of the element at as shown in Figure 3 The degrees of freedom of each node are the in-plane ents, u, v, the deflection, w, and the rotations θ_x , θ_y The rotation, θ_z was included make the analysis of thin-walled sections of any shape more convenient

ent stiffness matrix of advanced composite thin sections can be obtained through ps Firstly by adding the contribution of membrane actions Including the on of bending actions satisfies the second step The last step deals with the coupling membrane and bending actions

t Stiffness Matrix Due To Membrane Action

that the two displacement functions for u, v are first order polynomials as follows :

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \qquad (4\,a)$$

and

$$v = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy \qquad (4\,b)$$

quation 4 in a matrix form

$E]\ \{\alpha\}$ and then, $\{\alpha\} = [E]^{-1}\ \{\delta\}$ (5)

$\alpha\}$ is the vector of the unknown coefficients of the polynomials given in Equation (4) d can be expressed in terms of the u and v displacements of the nodes.

Equation (2 a) can be related at any point to displacements by differentiation and lation between strains and $\{\alpha\}$ can be written in a matrix form as follows

$$\begin{bmatrix} 0 & 1 & 0 & y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & x \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

Substituting Equation 5 into 6

$$\{\varepsilon^o\} = [Cp]\{\delta\} \qquad (7)$$

Where

[Cp] = Strain-displacement matrix due to membrane action

The relation between stresses and strains is

$$[\sigma] = [A]\{\varepsilon^o\}$$

$$[\sigma] = [A][Cp]\{\delta\} \qquad (8)$$

Applying the principle of virtual work to derive the element stiffness matrix by equating the internal work and the external work :

$$[Kp]_{8\times8} = t\int_s [Cp]^T [A][Cp]\,ds \qquad (9)$$

[Kp]= element stiffness matrix due to membrane actions

s = surface area of the element

2. Element Stiffness Matrix Due To Bending Action

Assuming that the displacement function is a third order polynomial function as follows :

$$w = \alpha_9 + \alpha_{10} x + \alpha_{11} y + \alpha_{12} z + \alpha_{13} xy + \alpha_{14} xz + \alpha_{15} yz + \alpha_{16} xyz + \alpha_{17} x^2 + \alpha_{18} x^2 y + \alpha_{19} x y^2 + \alpha_{20} y2 + \alpha_{21} x^3 + \alpha_{22} x^3 y + \alpha_{23} xy^3 + \alpha_{24} y^3$$

Curvatures and twist in Equation (2 b) are related at any point to displacements by differentiating the displacement function, w, to the second order and then the curvature displacement matrix [Cb] can be obtained in the same manner as in the membrane action

$$[Kb]_{16\times16} = t\int_s [Cb]^T [D][Cb]\,ds \qquad (10)$$

Stiffness Matrix Due To Coupling Between Membrane And Bending Actions

Element stiffness matrix due to coupling is obtained as follows

$$[Kc]_{16\times 8} = t\int_s [Cb]^T [B] [Cp]\, ds \qquad (11)$$

$$[Kc]_{8\times 16} = t\int_s [Cp]^T [D] [Cb]\, ds \qquad (12)$$

nt stiffness matrix due to coupling between membrane and bending actions

gy of GFRP Advanced Composites

ergy of an anisotropic laminated element can be written as

$$\int_V \left\{ \begin{bmatrix} \{\varepsilon^o\}^T \\ \{k\}^T \end{bmatrix} \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix} \{\varepsilon^o\} \\ \{k\} \end{bmatrix} \right\} dv \qquad (13\,a)$$

olume of the element

$$(\{\varepsilon^o\}^T[A]\{\varepsilon^o\} + 2\{\varepsilon^o\}^T[B]\{k\} + \{k\}^T[D]\{k\})\, ds \qquad (13\,b)$$

$\{\varepsilon^o\}$ = The strain energy due to membrane action

$\varepsilon\}$ = The strain energy due to coupling between membrane and bending action

$k\}$ = The strain energy due to bending action

valent Loads

l of the loads applied to the studied element can be expressed as

$$W = \int_s (X+Y+Z)\, ds \qquad (14)$$

, Y, and Z are the components of the loads applied to the studied element The

l potential of a layered system is

The element stiffness matrix [K] of order 24 x 24 can be obtained by applying the minimum total potential energy theorem. The full derivation of the element stiffness matrix is detailed elsewhere [3].

Transformation of Coordinates

The transformation matrix [λ] between the local and global coordinates systems shall be derived in order to analyze thin walled sections of any shape. The relationship between the displacements in local and global systems at node i of a typical element, shown in Figure 4, may be written in a matrix form as

$$\begin{bmatrix} u_i \\ v_i \\ w_i \\ \theta x_i \\ \theta y_i \\ \theta z_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 & 0 \\ 0 & -\sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \cos\beta & \sin\beta \\ 0 & 0 & 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{w}_i \\ \bar{\theta}x_i \\ \bar{\theta}y_i \\ \bar{\theta}z_i \end{bmatrix} \qquad (16)$$

In which u_i, v_i, w_i, θx_i, θy_i, θz_i = the reference surface displacements components of node i in the x, y, z direction, respectively, and $\bar{u}_i$, $\bar{v}_i$, $\bar{w}_i$, $\bar{\theta}x_i$, $\bar{\theta}y_i$, $\bar{\theta}z_i$ = the corresponding displacements in the X, Y, Z, direction. Equation (16) can be summarized as

$$[\delta] = [R][\bar{\delta}]$$

Applying the transformation matrix for the four nodes element, the element transformation matrix $[\lambda]_{24\times 24}$ can be obtained and then the element stiffness matrix $[\bar{K}]_{24\times 24}$ can be assembled in the global coordinate system,

$$[\bar{K}] = [\lambda]^T [K] [\lambda] \qquad (17)$$

EVALUATIVE ANALYSIS

The finite element formulation has been evaluated by analyzing the following problems. The first problem aims to check the accuracy of the model while the other problems show the

An isotropic thin-walled box section cantilever beam of length = 60" (152.4 cm), width = 18" (45.72 cm) and height = 12" (30.48 cm) - see Figure 5 - was analyzed using the developed model in order to check its accuracy. Tip load P = 5000 lb (2.3 ton) was applied at each corner of the beam as shown in Figure 5. Modulus of elasticity for the material of this beam was $30*10^6$ psi (2109.26 t/cm^2) and Poisson's ratio, ν=0.25. The finite element idealization consists of 24 nodes, and 20 elements. The degrees of freedom of each node equal six degrees of freedom, and the total degrees of freedom of the structure is 20*6=120. The tip deflection predicted by the proposed numerical model was compared with that obtained by the simple beam theory [13]. The predicted tip deflection was 0.0216105 " (0.0548906 cm) and that obtained by applying the simple beam theory [13] was 0.0204" (0.051816 cm). It is clear that the developed model gives an excellent agreement with the simple beam theory applied by Timoshenko and Goodier [13]. The behavior of some other structures was predicted successfully by the developed model and the results are detailed elsewhere [3]

Effect of Material Anisotropy (Moduli Ratio, E_{11} / E_{22})

A cantilever GFRP composite box section beam of length = 180 cm and breadth B = Height H = 20 cm was analyzed in order to assess the effect of changing moduli ratio for the material of such thin-walled section. The thickness of the beam section equals 1 cm and consists of three GFRP layers of fiber orientations (0° / 90° / 0°) of equal thickness. The beam was subjected to a load of 1.5 ton at the tip (0.75 ton at each corner). The material properties of the GFRP beams were as follows:

E_{11} / E_{22} had variable ratios = 2, 5, 10, 20, 30, 40, 50 ; G_{12} / E_{22} = 0.5 ; υ_{12} = 0.25

Where

E_{22} = 70.5 t / cm^2

E = Modulus of elasticity, G = Shear modulus, υ = Poisson's ratio

The subscripts (1) and (2) refer to the longitudinal and transverse directions of fibers respectively (See Figure 1).

Figure 6 shows the deflection results versus different ratios of E_{11} / E_{22} in a graphical form. It can be seen that an increase in the moduli ratio results in a significant decrease in the

deflection from 4.1 cm to 0.42 cm. This behavior may be attributed to the f stiffness of the studied section increases with an increase in the material anisotro ratio). This is in agreement with the expected behavior of the beam since a high mo indicative of higher modulus of elasticity in the longitudinal direction along which straining actions are exhibited. It is to be noted that an increase of the moduli rat value of about 25 provides very limited enhancement in the beam deflection.

Effect of Fibers Orientation Angle

The cantilever beam, studied above, was reanalyzed to assess the effect of cha orientation angle of the layers of such thin-walled box section on its behavior. section of the beam is of the same thickness and properties as in the previous prob cases were studied in this problem, the first one is a control case of top, bottom two webs with lay up of (0° / 90° / 0°). The second case had orientation angles o 0°) for the two webs and different orientation angles (0° / θ° / 0°) for top and bott The third one kept the orientation angles (0° / 90° / 0°) for the top and bottom fl the two webs had different orientation angles (0° / θ° / 0°). The last case conside fibers orientation angles are different (0° / θ° / 0°) for all four sides of the section. orientation angles were θ° = 15°, 25° 35° and 45°. The material properties o beams were as follows:

E_{11} / E_{22} = 40, G_{12} / E_{22} = 0.5, υ

Where

E_{11} = 2820 t / cm^2 and E_{22} = 70.5 t / cm^2

The deflection results for different fibers orientation angles are shown in Figure 7. expected, the control case, i.e. Case 1 showed no variation in deflections as orientations varied. Case 2 was shown to be the closest to the results of Case 1 al deflections tended to increase slightly with an increase in the angle of orientation cases, deflection values on the order of 0.47cm were noted. Cases 3 and 4, on the provided the better deflection results where both cases shared a common optimu fiber orientation that was in the vicinity of 30°. At the optimum angle, the beams e

veen values of tip deflections, namely 0.3 cm. Thus, cases 1 and 2 yielded 50% more
ns compared to cases 3 and 4. It can be argued that the contribution of the webs to
ness of the beam section is more than the contribution of top and bottom flanges.
Cases 3 and 4 had lower deflection for the fibers oriented closer to the longitudinal
e beam compared with Cases 1 and 2.

f Fiber Volume Fraction

section, the fiber-volume fraction of the four sides of the thin-walled box section beam
en as a variable instead of the fibers orientation angle. The change of fiber volume
in turn, affected the material properties as presented in Table 1 [14]. The fibers
n angle for the box-section beam was kept constant of lay up (0° / 90° / 0°). The
s studied previously were reanalyzed in order to assess the effect of fiber volume
on the behavior of thin-walled beams. The first case is considered as a control case,
s material properties as in the previously studied beam. The second one had the same
properties for the two webs and five different material sets of variable fiber volume
were considered for the top and bottom flanges. The proposed material sets were
om Ref. [14] and presented in Table 1. In the third case, the material properties were
stant for the top and bottom flanges while the two webs had the five different material
vn in Table 1. The fourth and last case considered that the five different material sets
Table 1 are applied to all four sides of the section.

erical results for tip deflection are shown in Figure 8. It can be seen from the figure
4 is the most one affected by changing the fiber-volume fraction. For example,
fiber volume fraction from 27% (material set 1) to 64% (material set 5) resulted in a
of deflection from 0.35 cm to 0.17 cm. Moreover, Figure 8 shows that cases 2 and 3
ed by the fiber-volume fraction more than their sensitivity to the change in fiber
n angle (see Figure 7). It is interesting to notice that the deflection values obtained
ng fibers orientation angle to 25° for Case 2 and between 25°-35° for Cases 3 and 4
re 7) were very close to those obtained by increasing the fiber-volume fraction to
terial Set 2) as shown in Figure 8. However, it should be noted that it is more
l to adjust fiber orientations than to increase the fiber-volume fraction.

Effect of Coupling between Membrane and Bending Actions

In the previously studied beams, the box-section was considered to consist of three symmetric layers of equal thickness. This lead to a balanced section with negligible coupling between membrane and bending actions. In order to study the effect of such coupling on the behavior of the studied box-section beam, the section was considered to be of the same thickness (1.0 cm) but to consist of two cross ply layers (0 / 90°) of equal thickness. The coupling effect was assessed by comparing the deflection results for different moduli ratios to those obtained by neglecting the coupling between membrane and bending actions, i.e. omitting the [B] matrix in Equation (13) from the element stiffness matrix. Figure 9 shows the results of the two cases along with the results of three layers (0° / 90° / 0°) shown in Figure 6. It can be seen from the figure that neglecting the coupling effect lead to an underestimation of the deflection results, which may, in turn, lead to a misleading approach for the design of such elements. However, the coupling effect vanishes with the decrease of moduli ratio. It is interesting to notice that the results of 3 symmetric layers (0° / 90° / 0°) are close to those of cross ply layers (0 / 90°) after neglecting coupling (orthotropic solution). This is probably due to the absence of coupling in the case of three (odd number) of symmetric layers. This condition is in essence analogous to forcing a beam where coupling is inherent to behave as though such coupling does not exist.

Effect of Layer Transposition

Figure 10 depicts the effect of layer transposition on the tip deflection for a range of moduli ratios. The basic trend of deflection reduction along with an increase in the moduli ratio remains evident, however, beams with 90° / 0° / 90° arrangement are observed to yield higher deflections as compared to identical beams with 0°/ 90° / 0° arrangement. This result signifies that longitudinal membrane action is predominant to transverse membrane action since the composite parameters of these cases are obtained by interchanging A_{11} and A_{22} with the remaining terms being the same. This is in agreement with the expected beam behavior where longitudinal stresses are predominant to transverse stresses.

In the case of angle ply arrangement, the above mentioned trend consistently prevails as may be seen in Figure 11 (a) through (d). A notable distinction in this figure, however, is that Case 1 (control) and Case 2 were the most affected by layer transposition. The layer transposition

increases with the increase of fiber orientation angle especially for Case 2. For example, ffect of layer transposition increased the deflection values for Case 2 by 27%, 33%, 36% 7% for fiber orientation angles of 15°, 25°, 35° and 45°, respectively. Despite the fact Cases 3 and 4 had lower deflection values than Cases 1 and 2, as explained above, they d less sensitivity to layer transposition.

2 presents the effect of layer transposition on the deflection for various percentages of olume fraction. Notable differences in the values of deflections were observed. For an increase of about 36 % in the deflection is detected with all five material sets ng the same deflection magnitudes. For Case 2, the increase is in the order of 28%, for Case 3 it is about 40% versus 30% for Case 4. A valuable observation for cases 2 4 is that deflections tend to decrease with an increase in the percentage of fiber fraction.

LUSIONS

he scope of this investigation, the following conclusions may be drawn:

inite element model developed in this paper is capable of predicting the behavior of valled beams in an accurate and efficient manner. The formulation included in the can be easily linked to any ready package thus providing a valuable numerical module analysis of a broad spectrum of related GFRP beam problems.

sing the moduli ratio (material anisotropy) leads to a great increase in the stiffness of alled GFRP beams of moduli ratios up to about 25, such an increase, however, is insignificant for higher ratios. Within the scope of this work, a moduli ratio of 25 is onsidered an optimum choice.

ng both of the fiber orientation angle and fiber volume fraction lead to a decrease in lection of the studied advanced composite thin-walled beams. However, increasing volume fraction is more costly than increasing the angle of fiber orientation.

g between membrane and bending actions softens the beam's resistance. Thus the ct of coupling is to reduce the effective stiffness of the studied section. The de of such reduction depends on the degree of material anisotropy (moduli ratio). It

is, therefore, recommended that judicious choice of fiber patterns be made in order to av such coupling whenever possible.

5. Layer transposition has an impact on deflection for those beams possessing extreme val of percentage of fiber volume fraction, i.e., highest and lowest, whereas beams possessi moderate volume fractions are insensitive to such a transposition.

6. Deflections consistently decreased when fibers are oriented closer to the longitudinal axis the beam. This behavior remains unchanged for cross ply laminates as well as angle laminates.

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Table 1 Material properties of unidirectionally glass-reinforced materials [14]

Material set	% of fiber volume fraction	E_{11} (t/cm^2)	E_{22} (t/cm^2)	G_{12} (t/cm^2)	υ_{12}
1	27	2190	434	161	0.31
2	39	3020	513	190	0.29
3	46	3504	575	213	0.28
4	57	4565	707	263	0.26
5	64	4750	429	399	0.25

Table 2 Effect of layer transposition and fiber volume fraction on the deflection of the studied box cantilever beam

erial	Case 1 (Control)		Case 2		Case 3		Case 4	
et	0° / 90° /0°	90°/0° /90°	0° / 90° /0°	90°/0° /90°	0° / 90° /0°	90°/0° /90°	0° / 90° /0°	90°/0° /90°
	0.47	0.73	0.51	0.71	0.32	0.54	0.36	0.52
	0.47	0.73	0.46	0.64	0.30	0.50	0.30	0.43
	0.47	0.73	0.41	0.57	0.28	0.46	0.24	0.35
	0.47	0.73	0.38	0.52	0.26	0.42	0.20	0.31
	0.47	0.73	0.36	0.48	0.24	0.39	0.17	0.26

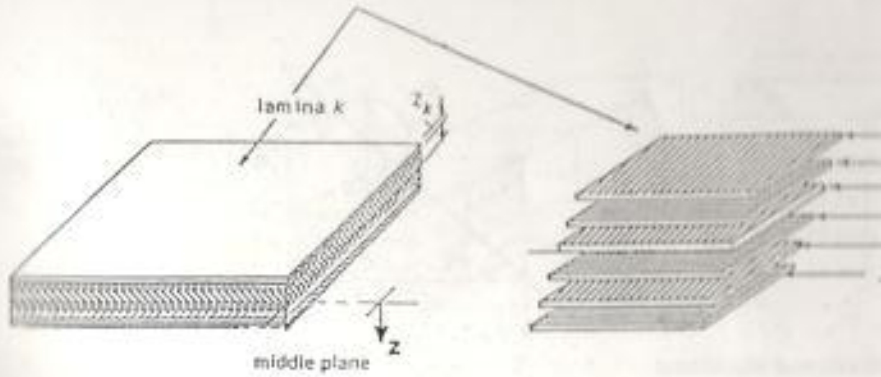


Figure 1 A typical advanced composite section with laminas of different orientati

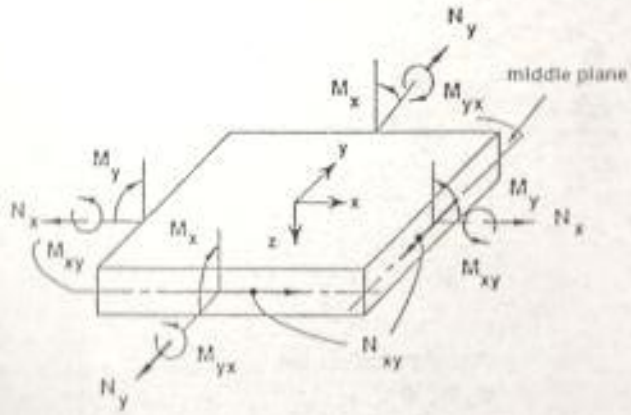


Figure 2 Stress resultants applied to a laminated advanced composite section

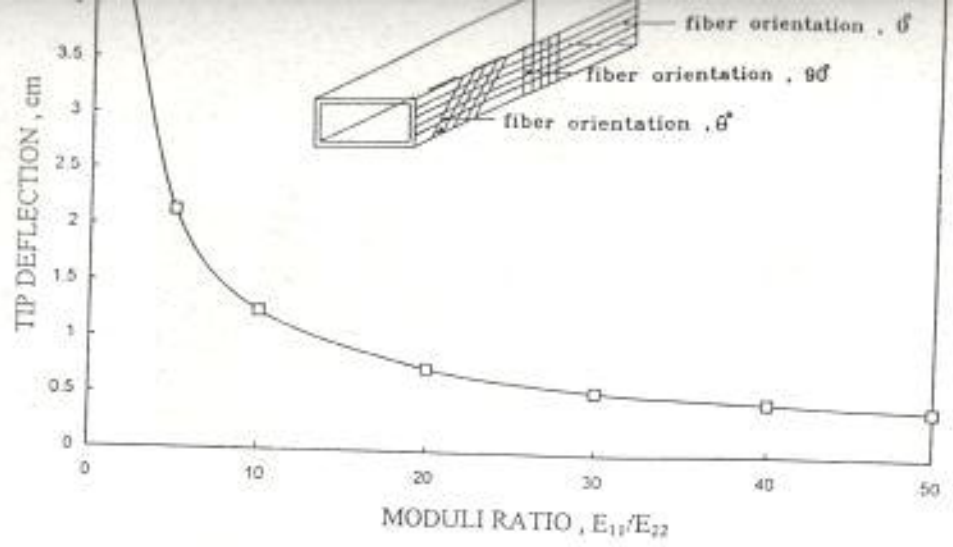


Figure 6 Effect of material anisotropy on the deflection of GFRP box cantilever beam.

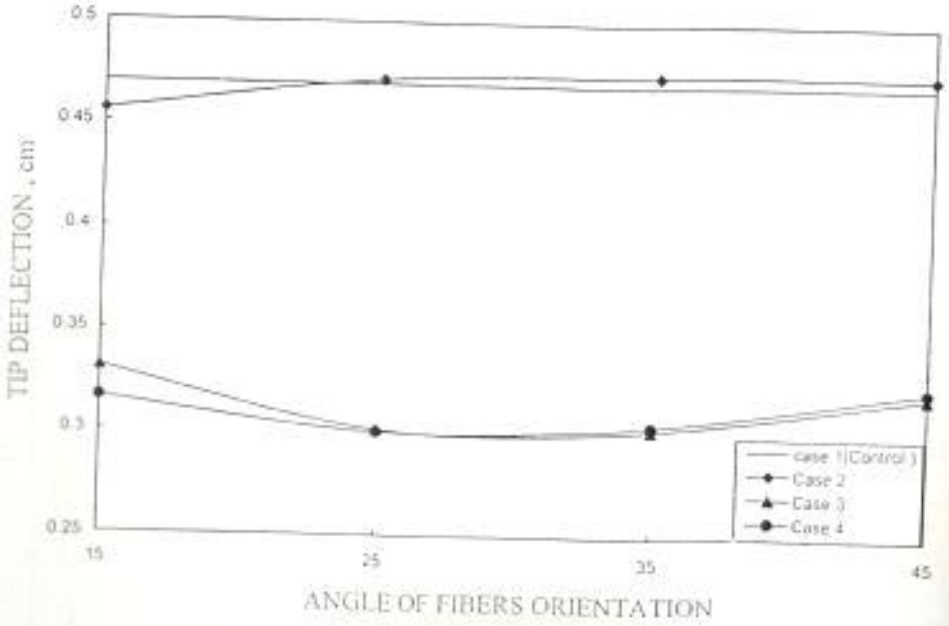


Figure 7 Effect of fibers orientation angle on the tip deflection of GFRP composite box-section beam.

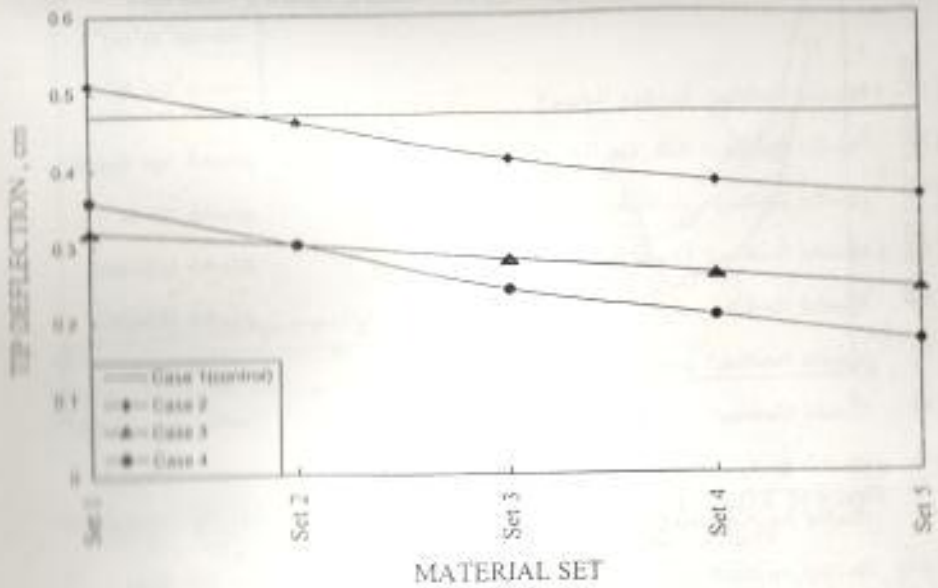


Figure 8 Effect of fiber-volume fraction on the tip deflection of GFRP box-section beam.

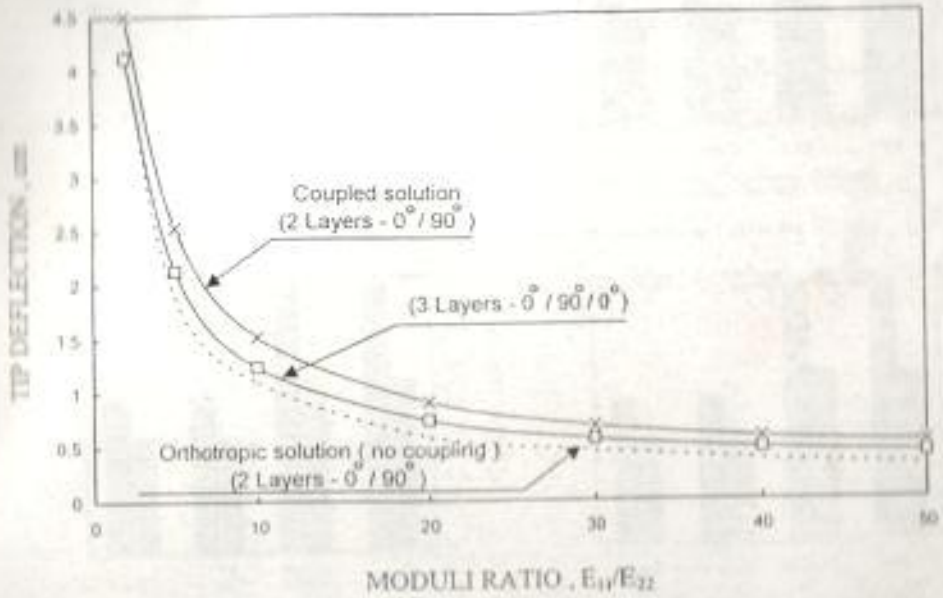


Figure 9 Effect of coupling on the behavior of box-section beam for different moduli ratios.

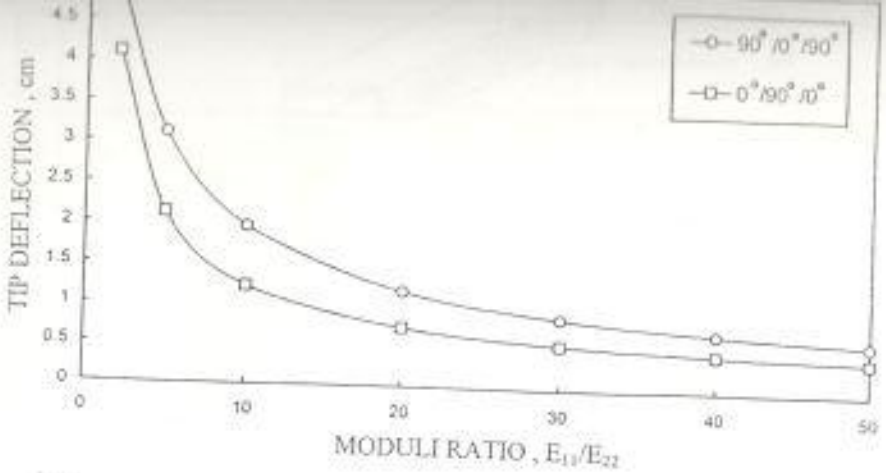
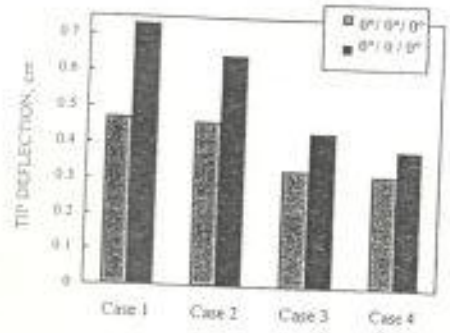
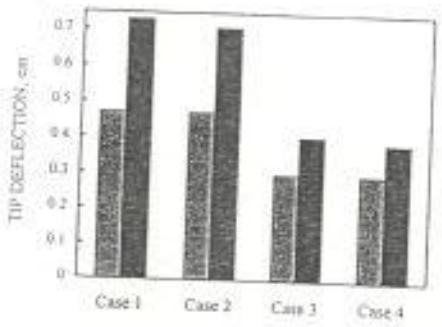


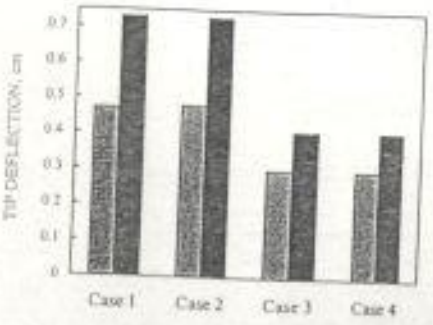
Figure 10 Effect of layers transposition on the deflection of GFRP box-section beam for different moduli ratios.



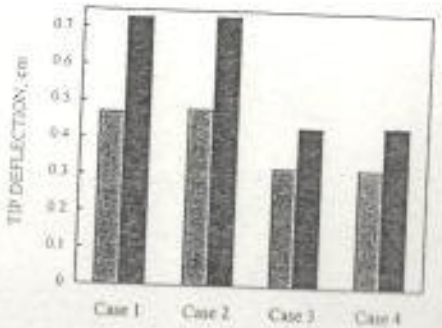
(a) Angle of fiber orientation (15°)



(b) Angle of fiber orientation (25°)



(c) Angle of fiber orientation (35°)



(d) Angle of fiber orientation (45°)

Figure 11 Effect of layers transposition on the deflection of the studied beam for different fiber orientations.

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(٦)	الأستاذ الدكتور/ محمود احمد مطاوع	جامعة الازهر
(٧)	الأستاذ الدكتور/ حمدي حامد شاهين	مركز بحوث البناء
(٨)	الأستاذ الدكتور/ احمد عاطف جاد الله	جامعة القاهرة
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(١١)	الأستاذ الدكتور/ محمد النبارى حماد	جامعة عين شمس
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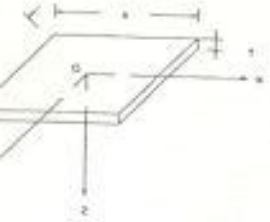
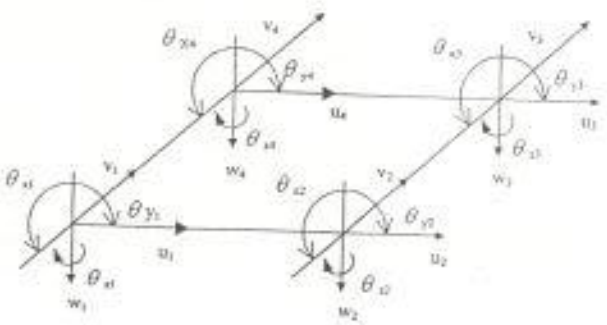
(١٤)	الأستاذ الدكتور/ احمد رجب	جامعة القاهرة
(١٥)	الأستاذ الدكتور/ علي عبد الرحمن	جامعة القاهرة
(١٦)	الأستاذ الدكتور/ محمد شوقي الغزالي	جامعة القاهرة
(١٧)	الأستاذ الدكتور/ محمود النقراشي عثمان	جامعة الازهر
(١٨)	الأستاذ الدكتور/ محمد محمد نصار	جامعة عين شمس
(١٩)	الأستاذ الدكتور/ عصام فواز	جامعة الازهر



ngular thin element with its local
obal axes



(b) In-plane and normal displacements

Figure 3 A rectangular element with six degrees of freedom per node

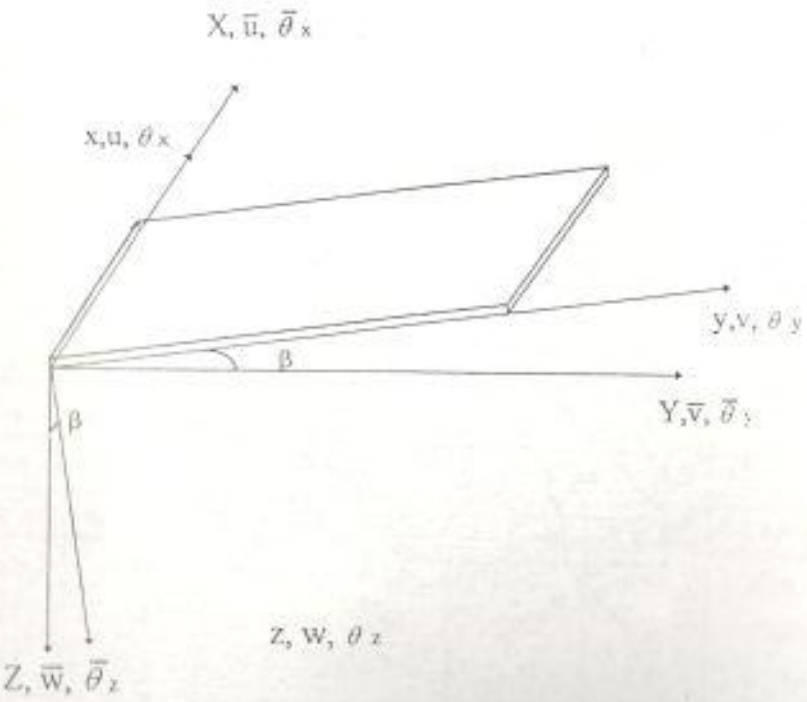
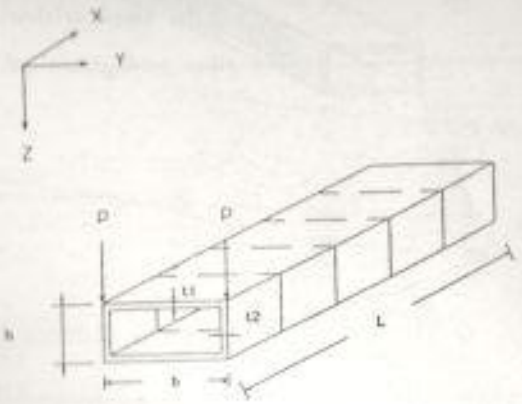
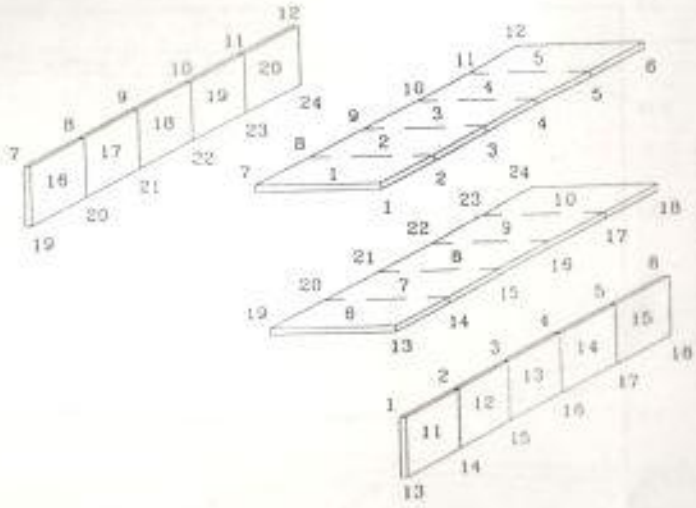


Figure 4 Individual and common coordinates systems



t1 = 1.0 inch (2.54 cm)
t2 = 0.5 inch (1.27 cm)
b = 18 inch (45.72 cm)
h = 12 inch (30.48 cm)

(a) Dimensions, loading conditions and mesh distribution



(b) Node and element numbering scheme

Figure 5 Finite element idealization of a box-section isotropic beam